Between the Pudding and the Pi - Outrageous Numbers

Musings for Christmas 2024
by Dr Ken Trethewey

Historical Prologue

Once upon a time, many, many, many ("How many?" "I dunno - a lot") years ago, there were twins called Romulus and Remus. As all good fabled twins were wont to do, they were constantly at loggerheads. One day, they got all their mates together and played a football match on a bit of rough ground down by the Tiber. Romulus and his friends called themselves Roma, and Remus's team was called Lazio. Roma beat Lazio v-nothing. Romulus had enjoyed himself so much (he'd scored a hat-trick and got an assist into the bargain) that he decided it was time to do something really big with his life instead of wasting it on football. In the changing rooms, straightening their togas, Romulus briefly consoled Remus by patting him on the back before winding him up.

"Hey, Rem, never mind bro. You were always weak at the back. You could do with a new central defender. All I had to do was nutmeg old Julius and it was in the back of the net. Tell you what - let's build a city. That'll cheer you up."

"Oh, really?" replied the downhearted Remus.

"Yeah. You've been building your place at the Palatine Hill and that's grown to quite a decent size, and I've got my new stadium coming along down here at Tiber Road. It accommodates MMCCCXX fans. If we bring our two premises together we can have quite a decent empire between us. One of us could be Caesar."

"Oh, Okay then," muttered Remus.

"How big is your place now?" asked Romulus, picturing a new, enlarged territory with himself in full command. After all, Remus was only central midfield whilst he, the great Romulus, was a striker and always got the limelight and the adulation.

"It's iii hectares," said Remus.

"Ah! That's a decent size, isn't it?

"Yes, I won it from those Greeks at Olympiakos. But then I thought of a problem. What if that gang came down from Inter Milan for their Champions' League away fixture and I lost it all in the after-match dust-up? Then what would I have?"

"You'd have nothing, dumbo."

"No, I don't mean that. You know I like numbers. We don't have a number for nothing."

Romulus didn't see the point at all. "You don't need a number for nothing."

"Yes you do!" argued Remus. "We've got i, ii, iii, iv, v, vi, vii, viii, ix and x to account for stuff. But if my place is iii hectares and I lose it to the Milanese, what's left?"

"Nothing, *nothing*, **nothing's left, you fool!**" shouted Romulus.

The argument was getting heated. Remus counterattacked.

"Yeah, null, nil, zero, zilch, zip, nowt, naught, bugger all, diddly squat - NOTHING! Plenty of words, but there's no symbol for it! We don't write the match scores as v-nothing. We need a symbol to put with the others," shouted Remus. "Those Arabs in Algeciras have got one - to them, the score would be v-0."

"Sod it. What do I care?" said Romulus, stomping off. Then he thought about his original idea once more. "Anyway, you haven't lost it, have you? You've got iii hectares and I've got ii hectares, so when we build our new city, which we can name after *my* team, between us we'll have ... er ... well ..."

Suddenly, Romulus looked embarrassed. "I never was any good at maths. What's ii hectares plus iii hectares?"

"Easy," said Remus. "It's v hectares."

"Oh, smart ass! How do you work that out then?"

Remus, who was pretty good with digits, quickly showed his brother how to do it with his fingers, but Romulus was still doubtful and called his brother a geek. Finally, Remus lost his temper. This latest family feud was developing nicely.

"You idiot, Romulus. It's the sum of the hat-trick you just scored, plus the penalties that the bent ref from Marseilles gave you in the second half. That makes v."

Romulus still didn't get it, but then he got really angry when he realised something rather crucial.

"That makes your plot bigger than my plot! I can't have that. It's my idea! I want to be Caesar. I'll just have to kill you ..."

The rest, as they say, is football history. The well-known fable ended in the way that all good stories of those years BC (Before Computers) did - a punch up. Sadly, Remus came off worst.

Back to Reality

It's still a mystery (to me at least) how the Romans got to be so good at things like building roads and installing central heating when they had such a woeful set of numbers. You see, in the beginning, it was all about allocating symbols to your fingers and thumbs. Someone - it wasn't Romulus, but could have been his great grandfather - looked at his dextra thumb (that's the one on his right hand because they spoke Latin in those days) and decided it had to be v for vectra. Then, feeling like he was on a roll, he decided his index finger should be i. (That's an abbreviation, d'you see?)

Feeling pleased with himself, this anonymous Roman mathematician decided that, as he moved along his hand, since there were now two fingers, he should use ii for his middle finger and iii for his ring finger. Cooking with gas now. We shall never know whether he thought that people would confuse their pinky with their ring finger if it was called iiii, or whether he simply ran out of inspiration and decided on iv. (What? The one before the thumb? No, surely not?) No logic prevailed, but it seemed like a good idea at the time.

Next he turned to his sinister hand. (That's left in Latin. Dextra was right and sinister was wrong.) Our friend was now in top gear, so his left thumb became x (for extra) and the remaining fingers vi, vii, viii and ix.

Brilliant! Ten fingers - ten numbers. He was now in business and ready to start charging his tenants higher rents. Little did he expect the trouble he would have when they changed from weekly to monthly payments. Taking off his shoes and using his toes would only have been a temporary fix.

Mathematicians might have preferred us to have evolved with just two fingers. It turned out to be the easiest form of counting - but you still needed a zero. The rest of us would have struggled to make cups of tea with only two fingers, but the maths guys would have loved it: 0, 1, 10, 11, 100, 101, 111, 1001, ... easy stuff. And just think how quickly we would have invented the computer. All our dates would have been different.

Curiously, there is nothing special about ten. Maths would have been just as easy if humans had evolved with eight fingers. Octopuses can count just as well as we can. The only difference is that there's no 8 and 9: 0, 1, 2, 3, 4 ,5, 6, 7, 10, 11, 12, 13, ... Just in case you haven't noticed, when you get to the end of the sequence of 2, 8 or 10, you just add a nought and start the pattern again.

So numbers are easy really. Well, that is, until you get past ten. My friend Joe pointed out to me a long time ago that we struggle to grasp the size of things when there are a few more than ten. If you saw a bowl with 17 apples in it, would you be able to guess *exactly* how many were there just by looking at it? How many of us would guess at 13, or perhaps 22? Maybe that doesn't matter too much. Maybe we should take the Romulan approach and be happy to know roughly how many. However, if you needed to buy 17 apples at Tesco, you wouldn't be happy just to pick up a handful. (Neither would Tesco.)

Which reminds me of the time when my dad asked one of his apprentices how many sixteenths there were in an inch? The lad, knowing that a sixteenth of an inch was pretty small, answered, "Crikey, there must be hundreds of them." The story illustrates perfectly how poor is our appreciation of the number of things when there are more than ten. We have to rely upon our intuition of size, which is poorly developed for most of us.

The Big And The Small

One of the daftest things I've noticed recently is to say that the new stadium at Tiber Road is so big it can be seen from space. What is that supposed to mean? Where from, and from how far away, and with what? When you think that my car number plate can be seen from orbiting satellites, well, enough said.

If I were to tell you that the height of a tree in my garden was 10 metres you might not have much idea until I said it was as tall as two double-decker buses stacked on top of each other. Hmmm. *Buses in my garden*? **On top of each other?**

If you wanted to tell your friend how far it was to the pub, he might feel more comfortable knowing it was only the length of two football pitches rather than 200 metres.

If I were to tell you that a customer of a water company had used 2,000 cubic metres of water you might have very little idea of how much that was until I helped you by saying that it's as much as it takes to fill an Olympic-size swimming pool. Oh, and what does that water weight? Why, two thousand tonnes, of course. Is that short tons or long tons? **No, it's two thousand tonnes**. Get it right!

It's a fun activity on Boxing Day after the movie has finished and before the cheese comes out to think of the many *faux* units of measurement that have crept into everyday language over decades.

And when it comes to something really, really, really small, we used to say that you couldn't slip a cigarette paper in the gap between a Scotsman and his wallet, but so few people now know just how narrow that is (the cigarette paper, that is) so we're stuck with the width of a human hair as our *faux* unit of smallness. Don't mind the fact that there might be such a thing as a micron.

I used to be a fan of F1 motor racing but I lost interest when these apparently amazingly fast cars appeared to go so slowly on my TV. (Odd that?) I found it curious that, when trying to differentiate between two cars, we needed to be told it was only sixteen one thousands of a second. Hey! How about saying sixteen milliseconds? The word millisecond is never used in F1 - now American owned, of course. And take a look at WRC to see how fast cars *really* go.

The blame for this rests squarely on the shoulders of Americans who, after abandoning everything associated with the British Empire at their tea party in 1776, and then screwing with our language, doggedly stuck with the British measurement system of feet, pounds, gallons, Fahrenheit etc. Even more daft still - when you realise it - is they are using the "Imperial" system of measurement invented by us Limeys. They quietly downplayed the accident that occurred when a European lunar probe crashed into the moon because Americans had made their contributions in miles instead of kilometres per hour. Anyway, I digress.

Let's stick with good old units of ten. At least the Americans don't count in sixteens - well, now I come to think of it, many do now because once we got past BC our years were called AD - After Decimalisation. (In truth, we shouldn't use AD any more because of the association with Christianity. Today we must use CE which stands for Computers Everywhere.)

Unless you are a fan of a football team like Plymouth Argyle, no-one counts with numbers based on ten anymore because we have moved entirely into the digital world of twos, the language of the computer, smart phone, payment card and AI.

Big Numbers Lurking Everywhere

I think I was about 8 years old when I heard this interesting puzzle. If you take a chess board and put a grain of rice on the first square, two grains on the second square, four on the third, eight on the fourth, and so on ... how much rice will you need to put on the 64th square? The answer is surprising. Being fascinated by puzzles as a kid, I soon realised *it was a lot*!

I have just entered the problem into Excel and it tells me the answer is 9,223,372,036,854,780,000 grains. That's crazy because I would not be able to tell you how many grains were in the bag of rice I just got from the supermarket.

Fortunately, I know how to estimate it because I'm a scientist. I just counted 72 grains in one gram of Tesco's best risotto rice, so that makes 72,000 grains in the 1 kg bag. Wow! That's a lot of bags of rice on that 64th square.

But in reality, I'm disappointed with Excel because I thought it would be more precise. There are fifteen digits of accuracy before the plethora of noughts on the end of the number tells me that the computer has approximated the answer. But I should be grateful because I just tried it a different way with computer programming and my laptop refused to go past 30 squares! The visual basic language I used is much less accurate. Now I'm even more disappointed.

A similar thing cropped up recently when I was briefly bored with life. I began to play Solitaire on my phone and wondered how long it would take before I started getting the same sequence of cards. I need not have worried for the number of different combinations for a pack of cards is crazily big. The answer (according to my rubbish Excel spreadsheet) is:

80,658,175,170,943,900,000,000,000,000,000, 000,000,000,000,000,000,000,000,000,000,000 ,000

(Again, there are 15 precise digits before the zeros kick in, so that has been approximated).

My AI friend tells me that the largest number I can calculate precisely on my Mac is 18,446,744,073,709,551,615, (20 precise digits) so Excel is selling me short by five digits. Now here's the thing. My computer is 64-bit, and this number should be the same answer as the number of grains of rice on the 64th square of the chess board? Check it out. Maybe I made a mistake? That's:

2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2

(Gulp!)

Do you trust AI? I need to revert to the longhand method I used at school. That will always work providing I live long enough to do the sums.

Never mind the Christmas Pudding -

What About the Pi?

Special methods must be used to calculate such large numbers exactly on a computer. You'll no doubt be relieved to know I will not report further on that. However, one of the more pointless computations has been the calculation of the ratio of the circumference of a circle to its diameter - that number we remember from school as pi (π). This is a very curious number that crops up everywhere. It is called an irrational number because, strangely, it can never be expressed exactly - the sequence of numbers after the decimal point literally never ends.

And not only does it never end, but there are no repeating sequences, despite there being six consecutive nines at position 762. At first commonly known as 22/7 or 3.142, the number has been continuously improved over the years so that in 2024 this number was calculated to an accuracy of 202 trillion decimals! I'm not going to print that one. According to NASA, it's unnecessary:

"... 37 decimal places are sufficient to calculate the circumference of the observable universe with an error no greater than the size of a hydrogen atom."

Who Needs Numbers Anyway?

We Have Computers Now.

The problem I'm getting to is that really big (and really small) numbers start to get difficult. We can't imagine just how big they are and most of us wouldn't know if the computer was right or not! The mathematicians tell us they have it all worked out, it would seem, so why doesn't my computer handle it?

Computers obviously began with 1s and 0s, but quickly moved on to work with 2s, 4s, 8s and 16s which was where I joined the game in the late 1970s. We said that the silicon chips were designed around 16-bit architecture. The transistors were about the size of a human hair - ha ha! (That's 75 microns actually. Today, they are a few nanometres in size.)

It was in the mid-90s when 32-bit software arrived with Windows 95 and only in the early 2020s when 64-bit computing was possible on home computers. The numbers on my computer could now be much bigger with extreme accuracy, but still clearly not big enough to handle outrageously large numbers.

You might be wondering why it matters to work with such large numbers? Well, Elon Musk needs to know exactly how much his 2025 salary should be and that is very important. When I was a kid, a billion was a million million and a trillion was a million million million - the bi = two and the tri = three made logical (British) sense.

Today, thanks to the Americans and Musk's every increasing salary, a billion is only (!) a thousand million and a trillion is only a million million, and as we watch our news bulletins we have come to accept these new definitions of very large numbers. As we think of any old billionaire and his money we may have lost the feeling of just how much money that is. But fear not because the computers can handle it...

I Need More Space

But when, on the odd occasion, we need to consider things of such great size, such as the distance Musk will need to send his rocket to reach Mars, it's not very convenient to have to write out a string of numbers as long as your arm. (I can just picture him sat in his office at SpaceX wrestling with the number of inches to Mars and how many (US) gallons of gas that will take. No, the maths guys have got it sorted with some funky notation that makes it easy - so THEY say! Going back to the chess board the number is written as 2^64 or even simpler 2^{64}.

We don't need to know how large it is exactly. We just know it's a very big number. We might even invent a new word for it such as ginormous, or humungous, or hyooooooooge! Now that's much easier than saying 10,347,683,456,223,657 bags of rice! Just call it 10^{15}.

I just asked my AI friend how far it was to Mars and this is the answer I got:

"The distance to Mars varies significantly depending on where Earth and Mars are in their orbits around the Sun. Here's a general breakdown:

- Closest approach (opposition): About 54.6 million kilometers (33.9 million miles). This happens roughly every 26 months.

- Average distance: About 225 million kilometers (140 million miles).

- Farthest distance: Around 401 million kilometers (249 million miles), when Mars is on the opposite side of the Sun from Earth."

What's Musk's email? He might find that useful. In fact, to do it with the minimum of fuel requires the involvement of gravity and what is called a

slingshot which takes much longer and the distance involved is phantasmagorical ... Sorry, I'm digressing again. Back to numbers.

The Biggest of the Big

So now we have started to think about space a lot more, and the ETs currently hovering over New Jersey in their UAPs have made us wonder how far they have come, we are starting to feel the need to work with some very, very, very large numbers. (Apparently the ETs came to find out how intelligent we are and have made careful note that all our videos of them have soundtracks of highly educated humans uttering strings of expletives too uncomfortable to be repeated here. I heard that one ET jotted down, "No, not ready yet.") But we think we ARE ready for them. After all we have these really large numbers now.

We are fairly confident that an alien intelligence has not arrived from our solar system. Big though that is, even we Earthlings have now sent rockets out across our solar system without noticing any other lifeforms. So the visitors in New Jersey (what's wrong with Cornwall?) must have come from the stars - another galaxy even.

Well this will make Musk's hair stand on end. The nearest star is four light years distant. How far is that? If I were American I'd enjoy telling you the number of inches, but it is four years of travelling at 299,792,458 meters per second which comes to 37,817,019,821,952 kilometres. And that's the closest star in our own galaxy.

It may have begun in 2000 when Google first appeared on my computer. Did you know that a googol is the number 10^{100}? That's an unimaginably large number - much larger than humungous. It's this big:

10,000,000,000,000,000,000,000,000,000,00
0,000,000,000,000,000,000,000,000,000,000,0
00,000,000,000,000,000,000,000,000,000,000,
000,000.

But it is a definite number. We call it finite. It's also more than the estimated number of sub-atomic particles in the Universe, a mere 10^{80}. And just in case you needed a larger number we have the googolplex which is 10^{googol}. That's ten with a googol of zeros after it. There isn't enough space in the Universe to write it down.

So what's the biggest, most outrageous number of all? Well oddly, we are quite familiar with it, yet we don't really understand just how big it is. It's infinity, written as ∞. Your first thought might be that there is nothing bigger than the Universe. Indeed, some people think the Universe is infinite, but it is not. And that piece of data I used just now - taking the smallest nuclear particle and finding the total in the Universe is still only 10^{80} - give or take a few quadrillion.

Let's just think about the numbers again - we call them the natural numbers: 1, 2, 3, 4, 5, 6, ... *there is no limit*. It does not have to refer to anything in the Universe, no matter how large it might become, because it's abstract and there is always a number at least one bigger than any number you can think of. So that's infinity.

Now here is a real puzzle. What about the numbers in between? Now we are back to that human hair thing once more, because between 1 and 2 we have 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8 and 1.9. So there are more numbers here than just the natural numbers alone. And what about the numbers between 1.1 and 1.2? Well, likewise, there are 1.11, 1.12, 1.13, ... you get the idea. And there is no limit to the thinness of the slices I want to make into the gaps between the natural numbers. (Jock's wallet is safe.) So if there are an infinity of natural numbers, then there are multiples of infinities of numbers in the gaps between them. Even the number we call infinity isn't big enough to describe all of the numbers available to us.

Romulus must be turning in his grave. "Outrageous!" I hear him mumble in Latin. And remember: A thousand of ANYTHING is A LOT!

Meanwhile, the Universe just gets on with its thing, not caring about numbers for a millisecond.

Postscript

I did the maths by hand!

The answer to the rice on the chessboard problem is:

9,223,372,036,854,775,808 grains.

The answer given by my AI friend is correct.

The largest number I can accurately calculate on my 64-bit computer is 2^{64} which I just found as :

18,446,744,073,709,551,615

This is double the rice answer but less 1!

Why? Because I started with 1 grain on the first square of the chessboard but $2^1 = 2$. And the difference of 1 is because we have to allow for the 0

I just heard Remus from the grave saying:

"I told you so!"